CLASS 10

SET-3

HINTS AND SOLUTIONS

1. **(D) :**

Letters	T	I	A	N	F	S	R	C	K
Codes	#	%	@	2	$	?	+	•	β

So, FANTASTIC will be coded as $@2#@?#%•.

2. **(D) :** From figure (1) to (2) : The element '•‿•' rotates 45° clockwise and is decreased by 1 and the arrow rotates 135° anti-clockwise and is increased by 1.

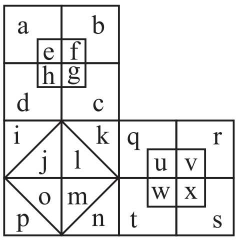
3. **(D) :** The rule followed is :

(17 + 85 + 68) ÷ 2 = 170 ÷ 2 = 85;

(19 + 67 + 58) ÷ 2 = 144 ÷ 2 = 72

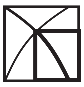
So, (38 + 86 + 66) ÷ 2 = 190 ÷ 2 = $\boxed{95}$

4. **(D) :**



Squares formed are: e, f, g, h, u, v, w, x, ae, bf, cg, dh, ij, kl, mn, op, qu, rv, sx, wt, jlmo, efgh, uvxw, ...*i.e.,* more than 22.

5. **(D) :**



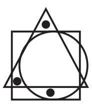
6. **(A) :** A. $J \xrightarrow{-3} G \xrightarrow{-3} D \xrightarrow{-2} B$

B. $U \xrightarrow{-4} Q \xrightarrow{-3} N \xrightarrow{-2} L$

C. $N \xrightarrow{-4} J \xrightarrow{-3} G \xrightarrow{-2} E$

D. $Y \xrightarrow{-4} U \xrightarrow{-3} R \xrightarrow{-2} P$

7. **(C) :**



8. **(C) :** 1, 6, 7 : Inner and outer figures are similar with shaded inner figure.

2, 5, 9 : Figures are divided into four parts.

3, 4, 8 : Contains two different figures one inside another with inner figure shaded.

9. **(B) :** Arranging the words in alphabetical order, we get, Forest, Forever, Forward, Fresher, Friend *i.e.,* 1, 3, 4, 2, 5.

10. **(C) :** Order of persons from the tallest to the shortest is, *M*, *T*, *D*, *R*, N. So, *M* is the tallest among all.

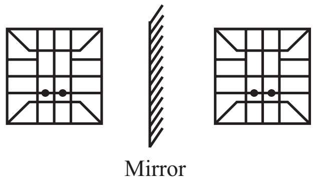
11. **(D) :** Letters on the opposite faces are : (K, P), (M, B) and (A, H).

12. **(B) :** 5P#6G@$\boxed{\text{F*5}}$%L8+E@7$\boxed{\text{R\#9}}$KM÷6GT

13. **(D)**

14. **(A) :** The man's father-in-law's wife is his mother-in-law. The only daughter of man's mother-in-law is his wife.

15. **(C) :**



16. **(D)**

17. **(B) :** We have, 5 cot θ = 3

$\Rightarrow \cot\theta = \frac{3}{5}$

$$\frac{5\sin\theta - 3\cos\theta}{4\sin\theta + 3\cos\theta}$$

Dividing both numerator and denominator by sin θ, we get

$$\frac{5\frac{\sin\theta}{\sin\theta} - 3\frac{\cos\theta}{\sin\theta}}{4\frac{\sin\theta}{\sin\theta} + 3\frac{\cos\theta}{\sin\theta}} = \frac{5-3\cot\theta}{4+3\cot\theta}$$

$$= \frac{5-3\left(\frac{3}{5}\right)}{4+3\left(\frac{3}{5}\right)} = \frac{5-\frac{9}{5}}{4+\frac{9}{5}} \qquad \left(\because \cot\theta = \frac{3}{5}\right)$$

$$= \frac{\frac{25-9}{5}}{\frac{20+9}{5}} = \frac{16}{29}$$

18. **(B) :**

Class-interval	x_i	f_i	$f_i x_i$
0 – 6	3	6	18
6 – 12	9	11	99
12 – 18	15	25	375
18 – 24	21	35	735
24 – 30	27	18	486
30 – 36	33	12	396
36 – 42	39	6	234
		113	2343

$$\text{Mean} = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{2343}{113} = 20.73$$

19. (B) : We have, $x = 5-2\sqrt{6}$

$$\therefore \quad \frac{1}{x}=\frac{1}{5-2\sqrt{6}}=\frac{1}{5-2\sqrt{6}}\times\frac{5+2\sqrt{6}}{5+2\sqrt{6}}$$

$$=\frac{5+2\sqrt{6}}{(5)^2-(2\sqrt{6})^2}=\frac{5+2\sqrt{6}}{25-24}=5+2\sqrt{6}$$

$$\therefore \quad x+\frac{1}{x}=5-2\sqrt{6}+5+2\sqrt{6}=10$$

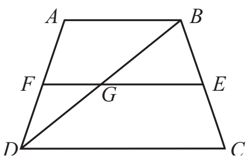
$$\Rightarrow \quad \left(x+\frac{1}{x}\right)^3=10^3$$

$$\Rightarrow \quad x^3+\frac{1}{x^3}+3\times x\times\frac{1}{x}\left(x+\frac{1}{x}\right)=1000$$

$$\Rightarrow \quad x^3+\frac{1}{x^3}+3\times 10=1000$$

$$\Rightarrow \quad x^3+\frac{1}{x^3} = 1000-30 = 970$$

20. (D) : In ΔDFG and ΔDAB,



$\angle DFG = \angle DAB$ (Corresponding angles)

$\angle FDG = \angle ADB$ (Common)

$\therefore \ \Delta DFG \sim \Delta DAB$ (*AA* similarity)

$$\Rightarrow \quad \frac{DF}{DA}=\frac{FG}{AB} \quad \ldots\text{(i)}$$

In trapezium *ABCD*,

$$\frac{AF}{FD}=\frac{BE}{EC} \quad (\because AB \parallel FE \parallel DC)$$

$$\Rightarrow \quad \frac{AF}{DF}=\frac{3}{4} \quad \left(\because \frac{BE}{EC}=\frac{3}{4}\ \text{(given)}\right)$$

Adding 1 to both sides, we get

$$\frac{AF}{DF}+1=\frac{3}{4}+1 \ \Rightarrow\ \frac{AF+DF}{DF}=\frac{7}{4} \ \Rightarrow\ \frac{AD}{DF}=\frac{7}{4}$$

$$\Rightarrow \quad \frac{DF}{AD}=\frac{4}{7} \quad \ldots\text{(ii)}$$

From (i) and (ii), $\frac{DF}{AD}=\frac{FG}{AB}=\frac{4}{7} \Rightarrow FG=\frac{4}{7}AB \ \ldots\text{(iii)}$

In ΔBEG and ΔBCD,

$\angle BEG = \angle BCD$ (Corresponding angles)

$\angle GBE = \angle DBC$ (Common)

$\therefore \ \Delta BEG \sim \Delta BCD$ (*AA* similarity)

$$\frac{BE}{BC}=\frac{EG}{CD} \quad \ldots\text{(iv)}$$

Now, $\frac{BE}{EC}=\frac{3}{4} \ \Rightarrow\ \frac{EC}{BE}=\frac{4}{3}$

Adding 1 to both sides, we get

$$\frac{EC+BE}{BE}=\frac{4+3}{3} \ \Rightarrow\ \frac{BC}{BE}=\frac{7}{3}$$

$$\Rightarrow \quad \frac{BE}{BC}=\frac{3}{7} \quad \ldots\text{(v)}$$

From (iv) and (v), $\frac{EG}{CD}=\frac{3}{7}$

$$\Rightarrow \quad EG=\frac{3}{7}CD=\frac{3}{7}\times 2AB \quad (\because CD = 2AB \text{ (given)})$$

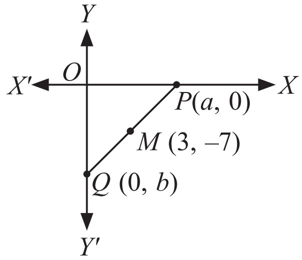
$$\Rightarrow \quad EG=\frac{6}{7}AB \quad \ldots\text{(vi)}$$

Adding (iii) and (vi), we get

$$FG+EG=\frac{4}{7}AB+\frac{6}{7}AB$$

$$\Rightarrow \quad FE=\frac{10}{7}AB \ \Rightarrow\ \frac{FE}{AB}=\frac{10}{7}$$

21. (D) : Let $P(a, 0)$ and $Q(0, b)$ be the points on x-axis and y-axis and M be the mid-point of PQ.



$\therefore$ Coordinates of M are

$$\left(\frac{a+0}{2},\frac{0+b}{2}\right)=\left(\frac{a}{2},\frac{b}{2}\right).$$

Given, coordinates of M are $(3, -7)$.

$$\therefore \quad \frac{a}{2}=3 \text{ and } \frac{b}{2}=-7 \ \Rightarrow a=6 \text{ and } b=-14$$

22. (B) : Perpendicular distance of the point $(-5, 6)$ from the x-axis is 6 units.

23. (B) : We are given that $x^2 + 9y^2 = 369$ and $xy = 60$

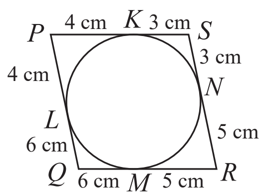
Consider, $(x - 3y)^2 = (x)^2 - 2(x)(3y) + (3y)^2$

$= x^2 - 6xy + 9y^2 = (x^2 + 9y^2) - 6xy$

$= 369 - 6 \times 60 = 9 = (3)^2$

$\Rightarrow \ x - 3y = 3 \quad (\because x > 3y, \text{ i.e., } x - 3y > 0)$

24. (B) : Here, four sides of the quadrilateral *PQRS* are tangent to the circle.



We know that, the lengths of tangents drawn from an external point to a circle are equal.

$\therefore \ QM = QL = 6$ cm, $RM = RN = 5$ cm

$SN = SK = 3$ cm and $PK = PL = 4$ cm

$\therefore \quad QM + MR = QR$

$\Rightarrow \quad QR = (6 + 5)$ cm = 11 cm

$PQ = PL + LQ$

$\Rightarrow \quad PQ = (4 + 6)$ cm = 10 cm

$SR = SN + NR = (3 + 5)$ cm = 8 cm

$PS = PK + KS = (4 + 3)$ cm = 7 cm

Now, perimeter of quadrilateral = $PQ + QR + RS + PS$

$= (10 + 11 + 8 + 7)$ cm

$= 36$ cm

Hence, the perimeter of quadrilateral is 36 cm.

25. **(B) :** Internal radius of sphere (r) = 2 cm
External radius of sphere (R) = 4 cm

Volume of hollow sphere $= \frac{4}{3}\pi(R^3 - r^3)$

$= \frac{4}{3}\pi(4^3 - 2^3)\text{ cm}^3$

$= \frac{4}{3}\times\frac{22}{7}\times 56\text{ cm}^3$

$= \frac{704}{3} = 234.67\text{ cm}^3$

26. **(D) :** Since, R is the mid-point of PQ and N is the mid-point of LM.

$\therefore PR = RQ$ and $LN = NM$

$PQ = PR + QR = 2\,PR$

$\Rightarrow PQ = 2\,NM \qquad (\because PR = NM)$

$\Rightarrow PQ = NM + NM = LN + NM$

$\Rightarrow PQ = LM$

27. **(A) :** Given, $a(b - c)x^2 + b(c - a)x + c(a - b) = 0$

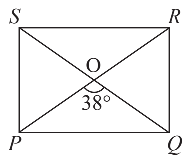
Let α be the other root of given equation.

Product of roots $= \alpha.1 = \frac{c(a-b)}{a(b-c)}$

$\Rightarrow \quad \alpha = \frac{c(a-b)}{a(b-c)}$

Hence, other root is $\frac{c(a-b)}{a(b-c)}$.

28. **(B) :** $\angle POQ + \angle QOR = 180°$ (Linear pair)



$\Rightarrow 38° + \angle QOR = 180°$

$\Rightarrow \angle QOR = 142°$

Since, diagonals of rectangle are equal and bisect each other.

$\therefore \ OR = OQ \Rightarrow \angle ORQ = \angle OQR$

In ΔQOR,

$\angle ORQ + \angle OQR + \angle QOR = 180°$

$\Rightarrow 2\,\angle ORQ + 142° = 180°$

$\Rightarrow \angle ORQ = \frac{38°}{2} = 19°$

Also, $\angle SRQ = 90°$ (Each angle of rectangle is 90°)

$\Rightarrow \angle SRO + \angle ORQ = 90°$

$\Rightarrow \angle SRO = 90° - 19° = 71°$

29. **(D) :** Let the larger number be x and the smaller number be y.

According to question,

$3x = 4y + 3$

$\Rightarrow \quad x = \frac{4y}{3} + 1 \qquad \ldots(i)$

and $7y = 5x + 1 \qquad \ldots(ii)$

Put value of x from (i) in (ii), we get

$\Rightarrow \quad 7y = 5\left(\frac{4y}{3}+1\right)+1 \quad \Rightarrow \quad 7y = \frac{20y}{3}+6$

$\Rightarrow \quad y\left(7-\frac{20}{3}\right) = 6 \quad \Rightarrow \quad y\left(\frac{1}{3}\right) = 6$

$\Rightarrow \quad y = 18$

Put value of y in (i), we get

$x = \frac{4(18)}{3} + 1 = 24 + 1 = 25$

Hence, larger number is 25 and smaller number is 18.

30. **(A) :** We have, first term = x, second term = y and last term = $2x$

$\Rightarrow$ Common difference = $y - x$

We know that,

$a_n = a + (n - 1)d$

Here, $a_n = 2x$, $a = x$, $d = y - x$

$\Rightarrow \quad 2x = x + (n - 1)(y - x)$

$\Rightarrow \quad (n-1) = \frac{2x - x}{y - x} = \frac{x}{y - x}$

$\Rightarrow \quad n = \frac{x}{y-x} + 1 = \frac{x + y - x}{y - x}$

$\Rightarrow \quad n = \frac{y}{y - x}$

$\therefore$ Sum of first n terms, $S_n = \frac{n}{2}\,[2a + (n - 1)d]$

Here, $n = \frac{y}{y-x}$, $a = x$, $(n-1) = \frac{x}{y-x}$, $d = y - x$

$S_n = \frac{y}{2(y-x)}\left[2x + \left(\frac{x}{y-x}\right)\times(y-x)\right]$

$= \frac{y}{2(y-x)}(2x + x) = \frac{3xy}{2(y-x)}$

31. **(C) :** In ΔBCA and ΔRPQ,

$BC = RP = 16$ cm (Given)

$\angle BCA = \angle RPQ = 60°$ (Given)

$CA = PQ = 15$ cm (Given)

$\therefore \quad \Delta BCA \cong \Delta RPQ$ (By SAS congruency)

So, $AB = QR$ (By CPCT)

$\Rightarrow \quad 5x + 4 = 7x - 4 \Rightarrow 2x = 8 \Rightarrow x = 4$

$\therefore \quad QR = 7x - 4 = 7 \times 4 - 4 = 24$ cm

32. **(A) :** Let r be the radius of circle.

Circumference of circle = 21.6 m

$\Rightarrow \quad 2\pi r = 21.6$ m

Length of arc = 5.4 m

$$\Rightarrow \quad \frac{\theta}{360^\circ} \times 2\pi r = 5.4$$

$$\Rightarrow \quad \frac{\theta}{360^\circ} \times 21.6 = 5.4$$

$$\Rightarrow \quad \theta = \frac{5.4 \times 360^\circ}{21.6}$$

$$\Rightarrow \quad \theta = 90^\circ$$

33. **(B) :** Putting $x = 3q - 19$ and $y = 6q - 17$ in $27x - 37y = -166$,

we get, $27(3q - 19) - 37(6q - 17) = -166$

$\Rightarrow 81q - 513 - 222q + 629 = -166$

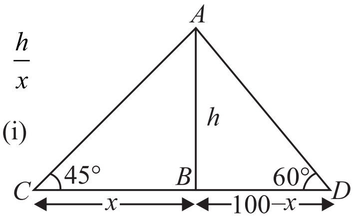
$\Rightarrow -141q + 116 = -166 \Rightarrow 141q = 282 \Rightarrow q = 2$

34. **(C) :** Let the height of the light house $AB = h$ m and C and D be position of boats.

$\therefore \quad CD = 100$ m

Let $CB = x$ m, then $BD = (100 - x)$ m

In ΔABC,



$$\tan 45^\circ = \frac{AB}{BC} \quad \Rightarrow \quad 1 = \frac{h}{x}$$

$\Rightarrow \quad x = h$ ...(i)

In ΔABD,

$$\tan 60^\circ = \frac{AB}{BD}$$

$$\Rightarrow \quad \sqrt{3} = \frac{h}{100 - x} \quad \Rightarrow \quad \sqrt{3}(100 - h) = h \quad (\because x = h)$$

$$\Rightarrow \quad 100\sqrt{3} - \sqrt{3}h = h \quad \Rightarrow \quad h + \sqrt{3}h = 100\sqrt{3}$$

$$\Rightarrow \quad h = \frac{100\sqrt{3}}{(1 + \sqrt{3})} \quad \Rightarrow \quad h = \frac{(100\sqrt{3})}{(\sqrt{3} + 1)} \times \frac{(\sqrt{3} - 1)}{(\sqrt{3} - 1)}$$

$$\Rightarrow \quad h = \frac{100\sqrt{3}(\sqrt{3} - 1)}{2} = 50\sqrt{3}(\sqrt{3} - 1) \text{ m}$$

35. **(C) :** Since, perpendicular from the centre of a circle to a chord of the circle, bisects the chord.

$\therefore \quad OQ \perp AB \quad \Rightarrow \quad AQ = QB$

In ΔAOQ, By Pythagoras theorem,

$AO^2 = OQ^2 + AQ^2 \quad \Rightarrow \quad 5^2 = 3^2 + AQ^2$

$\Rightarrow \quad AQ^2 = 25 - 9 = 16 \quad \Rightarrow \quad AQ = \sqrt{16} = 4$ cm

Length of tangents drawn from an external point to a circle are equal.

$\therefore \quad BQ = BP = 4$ cm

Also $OP \perp BC$

So, $BP = PC$

$\therefore \quad BC = BP + PC = (4 + 4)$ cm $= 8$ cm

36. **(A) :** Since, production increases uniformly by fixed number, it will form an A.P.

Now, production of mobile phones in 3rd year = 6000 units

$\Rightarrow \quad a_3 = 6000 \Rightarrow a + 2d = 6000$ ...(i)

Also, production of mobile phones in 5th year = 6500 units

$\Rightarrow \quad a_5 = 6500 \Rightarrow a + 4d = 6500$ ...(ii)

Subtracting (i) from (ii), we get

$2d = 500 \Rightarrow d = 250$

From (i), $a + 2 \times 250 = 6000 \Rightarrow a = 5500$

Hence, production in 8th year, $a_8 = a + 7d$

$= 5500 + 7 \times 250$

$= 5500 + 1750$

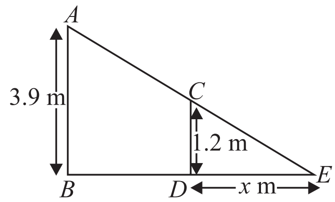
$= 7250$ units

37. **(B) :** Let lamp post is at point A, so $AB = 3.9$ m

Height of Sudha, $CD = 120$ cm $= 1.2$ m

Let the length of the shadow, DE be x m.

Distance BD walked by Sudha in 3 seconds $= 3 \times 1.5$ m $= 4.5$ m



In ΔABE and ΔCDE,

$\angle ABE = \angle CDE = 90^\circ$

$\angle BEA = \angle DEC$ (Common)

$\therefore \quad \Delta ABE \sim \Delta CDE$ (AA similarity)

$$\therefore \quad \frac{AB}{CD} = \frac{BE}{DE} \quad \Rightarrow \quad \frac{3.9}{1.2} = \frac{4.5 + x}{x}$$

$\Rightarrow \quad 3.9x = 1.2(4.5 + x) \quad \Rightarrow \quad 3.9x = 5.4 + 1.2x$

$$\Rightarrow \quad 3.9x - 1.2x = 5.4 \quad \Rightarrow \quad x = \frac{5.4}{2.7} = 2$$

Hence, the length of the shadow is 2 m.

38. **(D) :** A five rupee coin tossed three times.

$\Rightarrow$ Total number of outcomes $= 2 \times 2 \times 2 = 8$

Possible outcomes are : *HHH*, *HHT*, *HTT*, *HTH*, *TTT*, *THH*, *THT*, *TTH*

If Hussam win the game, then all tosses give the same result *i.e.*, *HHH*, *TTT*

In 6 chances when Hussam loses the game

i.e., HHT, HTT, HTH, THH, TTH, THT

$\Rightarrow$ Number of favourable outcomes = 6

$\therefore$ Required probability $= \frac{6}{8} = \frac{3}{4}$

39. (B) : Largest size of the tile = H.C.F. of 378 cm and 525 cm = 21 cm

40. (D) : Let $a = 3x$, $b = 4x$ and $c = 5x$

We have, perimeter of triangle = 48 cm

$\Rightarrow 3x + 4x + 5x = 48 \Rightarrow x = \frac{48}{12}$ cm = 4

$\therefore$ $a = 12$ cm, $b = 16$ cm and $c = 20$ cm

Semi-perimeter, $(s) = \frac{48}{2}$ cm = 24 cm

Area of triangle $= \sqrt{24\,(24-12)\,(24-16)\,(24-20)}$

$= \sqrt{24 \times 12 \times 8 \times 4} = \sqrt{12 \times 2 \times 12 \times 4 \times 2 \times 4}$

$= 96\text{ cm}^2$

41. (A) : Let the present age of son be x years and mother be y years.

Six years ago, son's age was $(x - 6)$ years and mother's age was $(y - 6)$ years.

Six years later, son's age will be $(x + 6)$ years and mother's age will be $(y + 6)$ years.

According to question,

$(y - 6) = 9(x - 6) \quad \Rightarrow \quad y = 9x - 48$ ...(i)

and $y + 6 = 3(x + 6) \quad \Rightarrow \quad y = 3x + 12$ ...(ii)

From (i), putting value of y in (ii), we get

$9x - 48 = 3x + 12$

$\Rightarrow 9x - 3x = 60 \Rightarrow 6x = 60 \Rightarrow x = 10$

Now, from (i) $y = 9x - 48 = 9(10) - 48 = 42$

Therefore, present age of son is 10 years and present age of mother is 42 years.

42. (B) : Let r be the radius of the hemispherical dome.

Curved surface area of dome $= 2\pi r^2$

$\therefore$ Cost of painting $= 2\pi r^2 \times 17$

According to question,

$\Rightarrow 83776 = 2 \times \frac{22}{7} \times r^2 \times 17$

$\Rightarrow r^2 = \frac{83776 \times 7}{2 \times 22 \times 17} = 784$

$\Rightarrow r = 28$ m

$\therefore$ Diameter = 2 × 28 m = 56 m

43. (B) : Let the two trains meet, x hours after 7 a.m.

Distance travelled by the train from station A in x hours $= (x \times 20)$ km

Distance travelled by the train from station B in $(x - 1)$ hours $= ((x - 1) \times 25)$ km

According to question,

$x \times 20 + (x - 1)25 = 110$

$\Rightarrow 20x + 25x - 25 = 110$

$\Rightarrow 45x = 135 \Rightarrow x = \frac{135}{45} = 3$

Therefore, the time at which two trains will meet is 3 hours after 7 a.m. *i.e.*, 10 a.m.

44. (D) : Let the principal be ₹ P and rate of interest be r %.

Difference of compound interest and simple interest for 2 years

$$= \left[P\left(1+\frac{r}{100}\right)^2 - P\right] - \frac{P \times r \times 2}{100}$$

$$= \left[P\left(1+\frac{r^2}{10000}+\frac{2r}{100}\right) - P\right] - \frac{2Pr}{100}$$

$$= P + \frac{Pr^2}{10000} + \frac{2Pr}{100} - P - \frac{2Pr}{100} = \frac{Pr^2}{10000}$$

Difference of compound interest and simple interest for 3 years

$$= \left[P\left(1+\frac{r}{100}\right)^3 - P\right] - \frac{P \times r \times 3}{100}$$

$$= \left[P\left(1+\frac{r^3}{1000000}+\frac{3r}{100}+\frac{3r^2}{10000}\right) - P\right] - \frac{3Pr}{100}$$

$$= \left[P+\frac{Pr^3}{1000000}+\frac{3Pr}{100}+\frac{3Pr^2}{10000} - P\right] - \frac{3Pr}{100}$$

$$= \frac{Pr^3}{1000000}+\frac{3Pr}{100}+\frac{3Pr^2}{10000}-\frac{3Pr}{100}$$

$$= \frac{Pr^3}{1000000}+\frac{3Pr^2}{10000} = \frac{Pr^2}{10000}\left(\frac{r}{100}+3\right)$$

Now, according to question,

$$\frac{\frac{Pr^2}{10000}\left(\frac{300+r}{100}\right)}{\frac{Pr^2}{10000}} = \frac{25}{8}$$

$$\Rightarrow \frac{300+r}{100} = \frac{25}{8}$$

$$\Rightarrow r = \frac{2500}{8} - 300 = \frac{100}{8} = 12\frac{1}{2}\%$$

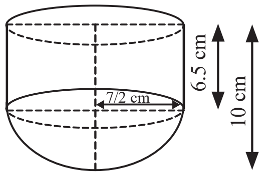
45. (B) : Let r be the radius of the hemispherical bowl and cylinder and h be the height of the cylinder.

$\therefore$ $r = \frac{7}{2}$ cm and $h = \left(10 - \frac{7}{2}\right)$ cm = 6.5 cm

Total capacity of the vessel = Volume of the cylinder

+ Volume of the hemisphere = $\pi r^2 h + \frac{2}{3}\pi r^3$

$= \pi r^2\left(h + \frac{2}{3}r\right)$



$= \frac{22}{7} \times \left(\frac{7}{2}\right)^2 \left(6.5 + \frac{2}{3} \times \frac{7}{2}\right)$

$= \frac{22 \times 7}{4} \times \frac{53}{6} = \frac{4081}{12} = 340.08 \text{ cm}^3$

46. (D) : P. False; We have, –3, $\frac{-1}{2}$, 2, ...

$d = \frac{-1}{2} + 3 = \frac{-1+6}{2}$

$\Rightarrow d = \frac{5}{2}$

Q. True; We have, 8, 3, –2, ...

$a = 8,\ d = 3 - 8 = -5$

$\therefore S_{19} = \frac{19}{2}[2 \times 8 + (19 - 1) \times (-5)]$

$= \frac{19}{2}(16 - 90) = \frac{19}{2} \times (-74)$

$= -703$

R. False; We have, $S_{14} = 1050$, $a = 10$, $n = 14$

$S_n = \frac{n}{2}[2a + 1\ (n-1)d]$

$\Rightarrow 1050 = \frac{14}{2}[2 \times 10 + (13)d]$

$\Rightarrow 1050 = 7\ [20 + 13d]$

$\Rightarrow 1050 = 140 + 91d$

$\Rightarrow 1050 - 140 = 91d$

$\Rightarrow 91d = 910$

$\Rightarrow d = \frac{910}{91} = 10$

$\therefore a_{20} = a + (n-1)d = 10 + (20 - 1) \times 10$

$= 10 + 19 \times 10 = 10 + 190$

$= 200$

47. (B) : Number of red balls = 11

Number of green balls = 19

Number of yellow balls = 29

Number of blue balls = 36

Number of orange balls = 15

Total number of balls = 11 + 19 + 29 + 36 + 15 = 110

(i) Number of balls which are not blue = 110 – 36 = 74

Number of favourable outcomes = 74

$\therefore$ So, required probability = $\frac{74}{110} = \frac{37}{55}$

(ii) Number of yellow balls = 29

$\therefore$ Number of favourable outcomes = 29

So, required probability = $\frac{29}{110}$

(iii) Number of balls which are not orange = 110 – 15

= 95

$\therefore$ Number of favourable outcomes = 95

So, required probability = $\frac{95}{110} = \frac{19}{22}$

48. (D) : Length of $PQ = \sqrt{(2+1)^2 + (7-1)^2} = \sqrt{3^2 + 6^2}$

$= \sqrt{9 + 36} = \sqrt{45} = 3\sqrt{5}$ units

Area of $PQRS = PQ \times QR = 15$ sq. units

$\Rightarrow 15 = 3\sqrt{5} \times QR \Rightarrow QR = \sqrt{5}$ units

Since, R lies on positive x-axis, let its coordinates be $(x_1, 0)$.

Length of $QR = \sqrt{5} = \sqrt{(x_1 + 1)^2 + (0 - 1)^2}$

$\Rightarrow \sqrt{5} = \sqrt{x_1^2 + 2x_1 + 2}$

Squaring both sides, we get

$5 = x_1^2 + 2x_1 + 2 \Rightarrow x_1^2 + 2x_1 - 3 = 0$

$\Rightarrow x_1^2 + 3x_1 - x_1 - 3 = 0$

$\Rightarrow x_1(x_1 + 3) - 1\ (x_1 + 3) = 0$

$\Rightarrow (x_1 + 3)\ (x_1 - 1) = 0$

$\Rightarrow (x_1 - 1) = 0$ or $(x_1 + 3) = 0$

$\therefore x_1 = 1$ ($\because x_1 \neq -3$ as x_1 lies on positive x-axis)

$\therefore$ Co-ordinates of R are (1, 0).

Now, $AP = PQ - QA = 3QA - QA$ ($\because PQ = 3QA$)

$\Rightarrow AP = 2QA$

So, $AQ : AP = 1 : 2$

Since A lies on the y-axis, let coordinates of A be $(0, y)$.

Using section formula, we have

$y = \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2} \Rightarrow y = \frac{1 \times 7 + 2 \times 1}{1 + 2}$

$\Rightarrow y = \frac{9}{3} = 3$

$\therefore$ Coordinates of A are (0, 3).

49. (B) : (i) Let x be the natural number.

According to question,

$x + \frac{1}{x} = \frac{37}{6} \Rightarrow \frac{x^2+1}{x} = \frac{37}{6} \Rightarrow 6x^2 + 6 = 37x$

$\Rightarrow 6x^2 - 37x + 6 = 0 \Rightarrow 6x^2 - 36x - x + 6 = 0$

$\Rightarrow 6x\ (x - 6) - 1\ (x - 6) = 0$

$\Rightarrow (x - 6)\ (6x - 1) = 0 \Rightarrow x = 6$ or $x = \frac{1}{6}$

$\therefore \quad x = 6 \qquad \left(x \neq \frac{1}{6}, \because x \text{ is a natural number}\right)$

(ii) Since, $x = \frac{3}{2}$ is a root of the equation $Kx^2 + x - 15 = 0$

$\therefore \quad K\left(\frac{3}{2}\right)^2 + \frac{3}{2} - 15 = 0 \Rightarrow \frac{9K}{4} = \frac{27}{2}$

$\Rightarrow \quad K = \frac{27}{2} \times \frac{4}{9} = 6$

(iii) Since, D = 40

$(2)^2 - 4 \times 1 \times (-m) = 40$

$\Rightarrow \quad 4 + 4m = 40$

$\Rightarrow \quad 4m = 36 \Rightarrow m = 9$

$\therefore \quad \sqrt{m} = 3$

50. (C) : (a) Length of tangents from an external point to the circle are equal.

$\therefore \quad PA = PB = 7$ cm

and $CQ = CA = 2.5$ cm

Now, $PC = PA - AC = (7 - 2.5)$ cm $= 4.5$ cm

(b) Let r be the radius of the given circle.

In ΔOTP, $\angle OTP = 90°$

($\because$ Tangent at any point of a circle is perpendicular to the radius through point of contact)

Now, in ΔPOT, by Pythagoras theorem,

$PO^2 = PT^2 + TO^2$

$\Rightarrow \quad (PA + AO)^2 = PT^2 + TO^2$

$\Rightarrow \quad (2 + r)^2 = 6^2 + r^2 \quad (\because AO = TO = \text{radii of the circle})$

$\Rightarrow \quad 4 + r^2 + 4r = 36 + r^2$

$\Rightarrow \quad 4r = 32$

$\Rightarrow \quad r = 8$ cm

(c) $\angle APB = 60° \quad \Rightarrow \quad \angle APO = 30°$

Now, in ΔOAP,

$$\sin 30° = \frac{OA}{OP} \Rightarrow \frac{1}{2}OP = 3\sqrt{3} \Rightarrow OP = 6\sqrt{3} \text{ cm}$$

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